Hidden Conformal Symmetry of the Reissner-Nordstrøm Black Holes

Chiang-Mei Chen*

Department of Physics and Center for Mathematics and Theoretical Physics, National Central University, Chungli 320, Taiwan

Jia-Rui Sun†

Department of Physics, National Central University, Chungli 320, Taiwan

(Dated: April 23, 2010)

Abstract

Motivated by recent progresses in the holographic descriptions of the Kerr and Reissner-Nordstrøm (RN) black holes, we explore the hidden conformal symmetry of nonextremal uplifted 5D RN black hole by studying the near horizon wave equation of a massless scalar field propagating in this background. Similar to the Kerr black hole case, this hidden symmetry is broken by the periodicity of the associated angle coordinate in the background geometry, but the results somehow testify the dual CFT description of the nonextremal RN black holes. The duality is further supported by matching of the entropies and absorption cross sections calculated from both CFT and gravity sides.

*Electronic address: cmchen@phy.ncu.edu.tw

†Electronic address: jrsun@phy.ncu.edu.tw

Contents

I. INTRODUCTION

In the past two years, the investigations on the holographic dual descriptions [1–5] for the black holes have archived substantial success, in particular for the Kerr [6–13] and the Reissner-Nordstrøm (RN) black holes [14–17], as well as the other generalizations [18–38]. The major progresses are made essentially on the extremal and near extremal limits in which the black hole near horizon geometries consist a certain AdS structure and the central charges of dual CFT can be obtained by analyzing the asymptotic symmetry following the approach in [39] or by calculating the boundary stress tensor of the 2D effective action as explicitly discussed in [40–44]. The supported evidences include the matching of the CFT and black hole entropies, and the agreement between the absorption cross section of a scalar field with the two point function of its dual operator.

It is naturally to expect that the holographic dual description should be legitimate for the general nonextremal black holes. However, there are no obvious hints for the AdS structure in the near horizon geometry of nonextremal black holes. The conformal symmetry somehow has been hidden and therefore the study of the corresponding dual CFT seems to be obscure. In the recent paper [45] an encouraging result has been shown that a massless scalar field is indeed able to probe the hidden conformal symmetry in the nonextremal Kerr black hole background, see also [46]. The work [45] is motivated by the fact that the near horizon wave

equation for a massless scalar field comprehend a conformal symmetry inherited from the back hole background geometry. In this approach, the dual CFT temperatures of left and right sectors can be identified. Moreover, the essential information of the operator dual to the scalar field can also be read out and the absorption cross section of the massless scalar field agrees with the two point function of its dual operator. However, in the background geometry, the conformal symmetry is broken to $U(1)_L \times U(1)_R$ due to the periodicity of the corresponding angle coordinate.

In this paper, we investigate the hidden conformal symmetry of RN black holes following the approach in [45]. The RN black holes can have a holographic dual CFT_2 description by uplifting to the 5D spacetime. The central charges generically depend on the radius of the extra dimension ℓ as: $c_L = c_R = 6q^3/\ell$. By analyzing the near horizon Klein-Gorden equation of a massless scalar field in the low frequency, $\omega \ll 1/m$ and low momentum $k/\ell \ll 1/m$ limit, we found that the wave equation can be derived from the Casimir operator of CFT_2. Consequently, the finite temperatures of left and right sectors are $T_L = \ell(2m^2-q^2)/2\pi q^3$ and $T_R = \ell m\sqrt{m^2-q^2}/\pi q^3$ respectively, and the CFT entropy precisely reproduces the black hole entropy

$$S_{\mathrm{CFT}} = \frac{\pi^2}{3}\frac{6q^3}{\ell}\left[\frac{\ell(2m^2-q^2)}{2\pi q^3} + \frac{\ell m\sqrt{m^2-q^2}}{\pi q^3}\right] = \pi r_+^2 = \frac{\mathrm{Area}}{4}.$$

We also check the agreement of the absorption cross section with the two point function of corresponding dual operator.

The outline of this paper is as follows. We review the uplifted RN black holes and discuss the associated properties in Section II. In Section III, we derive the near horizon wave equation for a massless scalar field and explore the hidden conformal symmetry in the equation. The CFT temperatures of left and right sectors are identified. We further, in Section IV, verify the absorption cross section of the massless scalar field and the two point function of its dual operator. Finally summarize our results in Section V.

II. UPLIFTED 5D RN BLACK HOLE

We first review the properties of 5D RN black hole uplifted from its 4D counterpart. Note that the electrically charged RN black hole in the 4D Einstein-Maxwell theory

$$I_4 = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\left(R - \frac{1}{4}F_{[2]}^2\right), \tag{1}$$

of metric and gauge potential ($m,\ q$ are mass and charge parameters)

$$
\begin{aligned}
ds_4^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2, \\
A_{[1]} &= -\frac{2q}{r}dt, \qquad f(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2},
\end{aligned} \tag{2}
$$

can be consistently uplifted to 5D Einstein-Maxwell theory

$$
I_5 = \frac{1}{16\pi G_5}\int d^5x\sqrt{-\hat{g}}\left(\hat{R} - \frac{1}{12}\hat{F}_{[3]}^2\right), \tag{3}
$$

via an inverse Kaluza-Klein (KK) reduction [17]

$$
ds_5^2 = \left(\ell d\chi + \frac{1}{2}A_\mu dx^\mu\right)^2 + ds_4^2, \qquad \hat{A}_{[2]} = \frac{\sqrt{3}}{2}A_{[1]} \wedge \ell d\chi. \tag{4}
$$

The explicit form of uplifted RN solution is

$$
\begin{aligned}
ds^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2 + \left(\ell d\chi - \frac{q}{r}dt\right)^2, \\
\hat{A}_{[2]} &= -\sqrt{3}\,\frac{q}{r}\,dt \wedge \ell d\chi.
\end{aligned} \tag{5}
$$

The radius of extra dimension of a circle, the parameter ℓ, is explicitly given in the solution to ensure the usual period of extra angle coordinate $\chi \sim \chi + 2\pi$. The gravitational constants in two different dimensions are related by $G_5 = 2\pi\ell G_4$. Hereafter we will assume $G_4 = 1$. A notable feature of the uplifted RN black hole is the emergence of the ergosphere at $-g_{tt} = 1 - 2m/r = 0$ which is an essential feature for superradiance study [17]. The corresponding black hole thermodynamic quantities, such as the Hawking temperature and the Bekenstein-Hawking entropy, are

$$
\begin{aligned}
T_H &= \frac{1}{4\pi}\frac{r_+ - r_-}{r_+^2}, \\
S_{BH} &= \frac{A_5}{4G_5} = \frac{A_4}{4} = \pi r_+^2,
\end{aligned} \tag{6}
$$

where $r_\pm = m \pm \sqrt{m^2 - q^2}$ are the black hole outer and inner horizon radius, respectively.

The dual CFT descriptions of the near extremal RN black hole have been studied both in the (warped) AdS_3/CFT_2 [14, 15, 17] and AdS_2/CFT_1 pictures [16]. The central charge of the CFT can be computed by analyzing the asymptotic symmetry of the near horizon geometry in the extremal limit. It is shown that, unlike the Kerr black hole case, the central charges for uplifted RN black holes depend on the choice of parameter ℓ. In particular, two

specific choices have been discussed: $c_L = c_R = 6q^3$ for $\ell = 1$ in [14, 15] and $c_L = c_R = 6q^2$ for $\ell = q$ in [16, 17]. Accordingly, the general expression for central charges can be written as

$$c_L = c_R = \frac{6q^3}{\ell}. \tag{7}$$

III. SCALAR FIELD EQUATION

Consider a bulk massless scalar field Φ propagating in the background of (5), the Klein-Gordon (KG) equation

$$\nabla_\alpha \nabla^\alpha \Phi = 0, \tag{8}$$

can be simplified by assuming the following form of the scalar field

$$\Phi(t, r, \theta, \phi, \chi) = \mathrm{e}^{-i\omega t + in\phi + ik\chi} S(\theta) R(r), \tag{9}$$

and reduces to two decoupled equations by separation of variables:

$$\partial_r(\Delta \partial_r R) + \left[\frac{(\omega r - kq/\ell)^2 r^2}{\Delta} - k^2/\ell^2 r^2 - \lambda_l \right] R \;=\; 0, \tag{10}$$

$$\frac{1}{\sin\theta} \partial_\theta(\sin\theta \partial_\theta S_l) + \left(\lambda_l - \frac{n^2}{\sin^2\theta} \right) S_l \;=\; 0, \tag{11}$$

where $\Delta = r^2 f = (r - r_+)(r - r_-)$. The angular equation simply implies that the separation constant should take the standard value $\lambda_l = l(l+1)$ for integer l and the solutions for S_l are just the standard spherical harmonic functions.

It is worth to note that the probe of a neutral massless scalar field in 5D uplifted RN black holes is equivalent to a probe in 4D RN black holes by a charged massive scalar field with mass $\bar{\mu}$ and charge e given by [17]

$$e = \frac{1}{2}\frac{k}{\ell}, \qquad \bar{\mu}^2 = \frac{k^2}{\ell^2}. \tag{12}$$

Therefore, the 5D momentum mode k generates both the charge and mass of the 4D scalar. The factor half is due to the k-momentum coupling only with the KK vector portion, namely half of the original 4D RN background charge.

Furthermore, the radial equation can be reformulated in the following form

$$\begin{aligned} \partial_r(\Delta \partial_r R) + \Bigg[& \frac{[(2mr_+ - q^2)\omega - (qr_+/\ell)k]^2}{(r - r_+)(r_+ - r_-)} - \frac{[(2mr_- - q^2)\omega - (qr_-/\ell)k]^2}{(r - r_-)(r_+ - r_-)} \\ & + (\omega^2 - k^2/\ell^2)(r^2 - q^2) + 2\omega(\omega m - kq/\ell)(r + 2m) \Bigg] R = l(l+1)R. \end{aligned} \tag{13}$$

We will see later that if the potential term in second line of Eq.(13) vanishes, then the remaining equation can be exactly obtained by the Casimir operator of the $SL(2,R)_L \times SL(2,R)_R$ Lie algebra. To accomplish this, we should require: (a) small frequency ω and momentum k/ℓ limit, namely $m\omega \ll 1$, $mk/\ell \ll 1$ (automatically implies $q\omega \ll 1$, $qk/\ell \ll 1$), and (b) near horizon limit of $r\omega \ll 1$ and $rk/\ell \ll 1$. Consequently, the near-horizon radial wave equation reduces to

$$\partial_r(\Delta\partial_r R) + \left[\frac{\left[(2mr_+ - q^2)\omega - (qr_+/\ell)k\right]^2}{(r-r_+)(r_+-r_-)} - \frac{\left[(2mr_- - q^2)\omega - (qr_-/\ell)k\right]^2}{(r-r_-)(r_+-r_-)}\right] R = l(l+1)R. \tag{14}$$

The two sets of symmetry generators of the AdS$_3$ space with radius L, in the Poincaré coordinates: $(w^\pm, y)$,

$$ds_3^2 = \frac{L^2}{y^2}(dy^2 + dw^+dw^-), \tag{15}$$

are

$$\begin{aligned} H_1 &= i\partial_+, \\ H_0 &= i\left(w^+\,\partial_+ + \frac{1}{2}y\,\partial_y\right), \\ H_{-1} &= i\left((w^+)^2\,\partial_+ + w^+ y\,\partial_y - y^2\,\partial_-\right), \end{aligned} \tag{16}$$

and

$$\begin{aligned} \bar{H}_1 &= i\partial_-, \\ \bar{H}_0 &= i\left(w^-\,\partial_- + \frac{1}{2}y\,\partial_y\right), \\ \bar{H}_{-1} &= i\left((w^-)^2\,\partial_- + w^- y\,\partial_y - y^2\,\partial_+\right), \end{aligned} \tag{17}$$

assembling two copies of the $SL(2,R)$ Lie algebra

$$[H_0, H_{\pm 1}] = \mp i H_{\pm 1}, \qquad [H_{-1}, H_1] = -2iH_0. \tag{18}$$

Thus the corresponding Casimir operator is

$$\mathcal{H}^2 = \bar{\mathcal{H}}^2 = -H_0^2 + \frac{1}{2}\left(H_1 H_{-1} + H_{-1}H_1\right) = \frac{1}{4}\left(y^2\,\partial_y^2 - y\,\partial_y\right) + y^2\,\partial_+\partial_-. \tag{19}$$

Converting the Poincaré coordinates $(w^\pm, y)$ to the coordinates (t, r, χ) of uplifted RN black hole by the following transformations

$$w^+ = \sqrt{\frac{r-r_+}{r-r_-}}\,\exp(2\pi T_R \chi + 2n_+ t),$$

$$
\begin{aligned}
w^- &= \sqrt{\frac{r-r_+}{r-r_-}}\,\exp(2\pi T_L\chi + 2n_- t),\\
y &= \sqrt{\frac{r_+-r_-}{r-r_-}}\,\exp[\pi(T_R+T_L)\chi + (n_+ + n_-)t],
\end{aligned}
\tag{20}
$$

where

$$
T_R = \frac{(r_+-r_-)m\ell}{2\pi q^3}, \qquad T_L = \frac{(r_++r_-)m\ell}{2\pi q^3} - \frac{\ell}{2\pi q}, \qquad n_\pm = -\frac{r_+\mp r_-}{4q^2}, \tag{21}
$$

we can directly calculate all the $SL(2,R)$ generators in terms of black hole coordinates

$$
\begin{aligned}
H_1 &= i\mathrm{e}^{-(2\pi T_R\chi+2n_+t)}\left(\sqrt{\Delta}\,\partial_r + \frac{m^2}{\pi q^2 T_R}\frac{r-q^2/m}{\sqrt{\Delta}}\,\partial_\chi + \frac{2T_L}{T_R}\frac{mr-\frac{m^2q^2}{2m^2-q^2}}{\sqrt{\Delta}}\,\partial_t\right),\\
H_0 &= i\frac{m^2}{\pi q^2 T_R}\,\partial_\chi + i\frac{2mT_L}{T_R}\,\partial_t,\\
H_{-1} &= i\mathrm{e}^{2\pi T_R\chi+2n_+t}\left(-\sqrt{\Delta}\,\partial_r + \frac{m^2}{\pi q^2 T_R}\frac{r-q^2/m}{\sqrt{\Delta}}\,\partial_\chi + \frac{2T_L}{T_R}\frac{mr-\frac{m^2q^2}{2m^2-q^2}}{\sqrt{\Delta}}\,\partial_t\right),
\end{aligned}
\tag{22}
$$

and

$$
\begin{aligned}
\bar{H}_1 &= -i\mathrm{e}^{-(2\pi T_L\chi+2n_-t)}\left(\sqrt{\Delta}\,\partial_r - \frac{2m^2-q^2}{2\pi q^2 T_L}\frac{r}{\sqrt{\Delta}}\,\partial_\chi - \frac{2mr-q^2}{\sqrt{\Delta}}\,\partial_t\right),\\
\bar{H}_0 &= i\frac{2m^2-q^2}{2\pi q^2 T_L}\,\partial_\chi + i2m\,\partial_t,\\
\bar{H}_{-1} &= -i\mathrm{e}^{2\pi T_L\chi+2n_-t}\left(-\sqrt{\Delta}\,\partial_r - \frac{2m^2-q^2}{2\pi q^2 T_L}\frac{r}{\sqrt{\Delta}}\,\partial_\chi - \frac{2mr-q^2}{\sqrt{\Delta}}\,\partial_t\right).
\end{aligned}
\tag{23}
$$

Finally the Casimir operator becomes

$$
\mathcal{H}^2 = \partial_r\Delta\partial_r - \frac{\left[(2mr_+-q^2)\partial_t + (qr_+/\ell)\partial_\chi\right]^2}{(r-r_+)(r_+-r_-)} + \frac{\left[(2mr_--q^2)\partial_t + (qr_-/\ell)\partial_\chi\right]^2}{(r-r_-)(r_+-r_-)}. \tag{24}
$$

Therefore, the near-horizon wave equation Eq.(14) can be formulated as

$$
\bar{\mathcal{H}}^2\Phi = \mathcal{H}^2\Phi = l(l+1)\Phi, \tag{25}
$$

and the conformal weights of dual operator of the massless field Φ should be

$$
(h_L, h_R) = (l+1, l+1). \tag{26}
$$

The microscopic entropy of the dual CFT can be computed by the Cardy formula which matches with the black hole Bekenstein-Hawking entropy

$$
S_{\mathrm{CFT}} = \frac{\pi^2}{3}(c_L T_L + c_R T_R) = \pi(2mr_+ - q^2) = \pi r_+^2 = S_{\mathrm{BH}}. \tag{27}
$$

The background geometry of the nonextremal uplifted RN black holes, however, does not consist the full $SL(2,R)_L \times SL(2,R)_R$ symmetry. Actually, this conformal symmetry is broken down to $U(1)_L \times U(1)_R$ by the periodicity of the angle coordinate χ. The $SL(2,R)$ generators are not periodic under the angular identification $\chi \sim \chi + 2\pi$, but are transformed to

$$w^+ \sim \mathrm{e}^{4\pi^2 T_R}\, w^+, \qquad w^- \sim \mathrm{e}^{4\pi^2 T_L}\, w^-, \qquad y \sim \mathrm{e}^{2\pi^2 (T_L+T_R)}\, y, \tag{28}$$

which just corresponding to operation of the $U(1)_L \times U(1)_R$ group element in $SL(2,R)_L \times SL(2,R)_R$

$$\mathrm{e}^{-i4\pi^2 (T_L \bar{H}_0 + T_R H_0)}. \tag{29}$$

IV. SCATTERING

The scattering process of a massive scalar field in the uplifted RN black hole background (5) has been discussed in [17] in the near extremal limit, and the absorption cross section calculated from the geometric side matches with the microscopic greybody factor calculated from the dual CFT side. In this section, we will extend the computation into nonextremal limit of the uplifted RN black hole. The near horizon region solutions, for $r \ll 1/\omega$, $r \ll \ell/k$, of Eq.(14) include both ingoing and outgoing modes

$$\begin{aligned} R^{(\mathrm{in})} &= \left(\frac{r-r_+}{r-r_-}\right)^{-i\frac{r_+(2m\omega - q\mu\omega - q\tilde{k})}{r_+-r_-}} (r-r_-)^{-l-1} \qquad (30)\\ &\quad F\left(1+l-\frac{2(2m^2\omega - q^2\omega - mq\tilde{k})}{r_+-r_-}, 1+l-i(2m\omega - q\tilde{k}); 1-i\frac{2r_+(2m\omega - q\mu\omega - q\tilde{k})}{r_+-r_-}; \frac{r-r_+}{r-r_-}\right), \\ R^{(\mathrm{out})} &= \left(\frac{r-r_+}{r-r_-}\right)^{i\frac{r_+(2m\omega - q\mu\omega - q\tilde{k})}{r_+-r_-}} (r-r_-)^{-l-1} \qquad (31)\\ &\quad F\left(1+l+\frac{2(2m^2\omega - q^2\omega - mq\tilde{k})}{r_+-r_-}, 1+l+i(2m\omega - q\tilde{k}); 1+i\frac{2r_+(2m\omega - q\mu\omega - q\tilde{k})}{r_+-r_-}; \frac{r-r_+}{r-r_-}\right), \end{aligned}$$

where $\tilde{k} = k/\ell$ and $\mu = q/r_+$ is the chemical potential. At the outer boundary of the matching region $r \gg m$ (but still $r \ll 1/\omega$, $r \ll 1/\tilde{k}$)

$$R^{(\mathrm{in})}(r \gg m) \sim A r^l + B r^{-l-1}, \tag{32}$$

with

$$A = \frac{\Gamma\left(1 - i\frac{2r_+(2m\omega - q\mu\omega - q\tilde{k})}{r_+ - r_-}\right)\Gamma(1+2l)}{\Gamma\left(1+l-i(2m\omega - q\tilde{k})\right)\Gamma\left(1+l-i\frac{2(2m^2\omega - q^2\omega - mq\tilde{k})}{r_+ - r_-}\right)}, \tag{33}$$

$$B = \frac{\Gamma\left(1 - i\frac{2r_+(2m\omega - q\mu\omega - q\tilde{k})}{r_+ - r_-}\right)\Gamma(-1-2l)}{\Gamma\left(-l-i(2m\omega - q\tilde{k})\right)\Gamma\left(-l-i\frac{2(2m^2\omega - q^2\omega - mq\tilde{k})}{r_+ - r_-}\right)}, \tag{34}$$

up to an overall constant independent of ω and k. Formally we should also find the asymptotic region solutions, for $r \gg m$, and then match two sets of solutions at the overlap region, i.e. $m \ll r \ll (1/\omega, 1/\tilde{k})$, in order to get the relations among the integration constants [9, 17]. However, the essential properties of the absorption cross section is indeed captured by the coefficient A such as

$$\begin{aligned} P_{\text{abs}} &\sim |A|^{-2} \\ &\sim \sinh\left(\frac{2\pi r_+(2m\omega - q\mu\omega - q\tilde{k})}{r_+ - r_-}\right) \\ &\quad \left|\Gamma\left(1+l-i(2m\omega - q\tilde{k})\right)\right|^2 \left|\Gamma\left(1+l-i\frac{2(2m^2\omega - q^2\omega - mq\tilde{k})}{r_+ - r_-}\right)\right|^2. \end{aligned} \tag{35}$$

To see explicitly that P_{abs} matches with the microscopic greybody factor of the dual CFT, we need to identify the related parameters. From the first law of black hole thermodynamics

$$T_H \delta S_{BH} = \delta m - \mu \delta q. \tag{36}$$

one can compute the conjugate charges as

$$\delta S_{BH} = \delta S_{CFT} = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R}, \tag{37}$$

and the solution is

$$\begin{aligned} \delta E_L &= \frac{(2m^2 - q^2)(2m\ell\delta m - q\delta q)}{q^3}, \\ \delta E_R &= \frac{2(2m^2 - q^2)m\ell\delta m - 2m^2 q\delta q}{q^3}. \end{aligned} \tag{38}$$

Therefore, the frequencies of left and right sectors can be read out by identifying $\omega = \delta m$, $k = \delta q$, namely

$$\begin{aligned} \omega_L &= \delta E_L = \frac{(2m^2 - q^2)(2m\ell\omega - qk)}{q^3}, \\ \omega_R &= \delta E_R = \frac{2(2m^2 - q^2)m\ell\omega - 2m^2 qk}{q^3}. \end{aligned} \tag{39}$$

Finally, the absorption cross section can be expressed as

$$P_{\rm abs} \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh\left(\frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R}\right) \left|\Gamma\left(h_L + i\frac{\omega_L}{2\pi T_L}\right)\right|^2 \left|\Gamma\left(h_R + i\frac{\omega_R}{2\pi T_R}\right)\right|^2, \tag{40}$$

which is the finite temperature absorption cross section for a 2D CFT.

V. CONCLUSION

The $\mathrm{AdS}_3/\mathrm{CFT}_2$ description for the RN black holes can be discussed by considering their uplifted counterparts. The central charges of the dual CFT can be computed by analyzing the near horizon geometry in the extremal limit. It turns out that the central charges will depend on the embedding, namely depend on the radius of the extra dimension. The holographic dual descriptions of RN black holes have been investigated for the extremal [14–16] and near extremal [17] limits. For the general nonextremal case, however, the geometry does not consist obvious AdS structure. Actually, the conformal symmetry is broken by the periodicity of the corresponding angle coordinate. Nevertheless, we found that a massless scalar field can probe this hidden conformal symmetry in the near horizon region, and by a suitable coordinates identification, the Casimir operator of $SL(2,R)_L \times SL(2,R)_R$ reproduces the Klein-Gorden equation of the massless scalar field in certain limits. Therefore we can derive the general expressions for the dual CFT temperatures of left and right sectors and the Cardy formula for the CFT entropy precisely agree with the black hole entropy. Moreover, we also can identify the conformal weights and frequencies of the operator dual to the massless scalar field. The further support for the holographic duality is the evidence that the absorption cross section computed from the gravity side agrees with the two point function of operators in the CFT.

Acknowledgement

This work was supported by the National Science Council of the R.O.C. under the grant NSC 96-2112-M-008-006-MY3 and in part by the National Center of Theoretical Sciences (NCTS).

[1] G. 't Hooft, "Dimensional reduction in quantum gravity," arXiv:gr-qc/9310026.

[2] L. Susskind, "The world as a hologram," J. Math. Phys. **36**, 6377 (1995) [arXiv:hep-th/9409089].

[3] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].

[4] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109].

[5] E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].

[6] M. Guica, T. Hartman, W. Song and A. Strominger, "The Kerr/CFT correspondence," Phys. Rev. D **80**, 124008 (2009) [arXiv:0809.4266 [hep-th]].

[7] O. J. C. Dias, H. S. Reall and J. E. Santos, "Kerr-CFT and gravitational perturbations," JHEP **0908**, 101 (2009) [arXiv:0906.2380 [hep-th]].

[8] Y. Matsuo, T. Tsukioka and C. M. Yoo, "Another Realization of Kerr/CFT Correspondence," Nucl. Phys. B **825**, 231 (2010) [arXiv:0907.0303 [hep-th]].

[9] I. Bredberg, T. Hartman, W. Song and A. Strominger, "Black Hole Superradiance From Kerr/CFT," JHEP **1004**, 019 (2010) [arXiv:0907.3477 [hep-th]].

[10] A. J. Amsel, D. Marolf and M. M. Roberts, "On the Stress Tensor of Kerr/CFT," JHEP **0910**, 021 (2009) [arXiv:0907.5023 [hep-th]].

[11] T. Hartman, W. Song and A. Strominger, "Holographic Derivation of Kerr-Newman Scattering Amplitudes for General Charge and Spin," JHEP **1003**, 118 (2010) [arXiv:0908.3909 [hep-th]].

[12] A. Castro and F. Larsen, "Near Extremal Kerr Entropy from AdS_2 Quantum Gravity," JHEP **0912**, 037 (2009) [arXiv:0908.1121 [hep-th]].

[13] M. Cvetic and F. Larsen, "Greybody Factors and Charges in Kerr/CFT," JHEP **0909**, 088 (2009) [arXiv:0908.1136 [hep-th]].

[14] T. Hartman, K. Murata, T. Nishioka and A. Strominger, "CFT duals for extreme black holes," JHEP **0904**, 019 (2009) [arXiv:0811.4393 [hep-th]].

[15] M. R. Garousi and A. Ghodsi, "The RN/CFT Correspondence," Phys. Lett. B **687**, 79 (2010) [arXiv:0902.4387 [hep-th]].

[16] C. M. Chen, J. R. Sun and S. J. Zou, "The RN/CFT Correspondence Revisited," JHEP **1001**, 057 (2010) [arXiv:0910.2076 [hep-th]].

[17] C. M. Chen, Y. M. Huang and S. J. Zou, "Holographic Duals of Near-extremal Reissner-Nordstrom Black Holes," JHEP **1003**, 123 (2010) [arXiv:1001.2833 [hep-th]].

[18] K. Hotta, Y. Hyakutake, T. Kubota, T. Nishinaka and H. Tanida, "The CFT-interpolating Black Hole in Three Dimensions," JHEP **0901**, 010 (2009) [arXiv:0811.0910 [hep-th]].

[19] H. Lu, J. Mei and C. N. Pope, "Kerr/CFT Correspondence in Diverse Dimensions," JHEP **0904**, 054 (2009) [arXiv:0811.2225 [hep-th]].

[20] T. Azeyanagi, N. Ogawa and S. Terashima, "Holographic Duals of Kaluza-Klein Black Holes," JHEP **0904**, 061 (2009) [arXiv:0811.4177 [hep-th]].

[21] D. D. K. Chow, M. Cvetic, H. Lu and C. N. Pope, "Extremal Black Hole/CFT Correspondence in (Gauged) Supergravities," Phys. Rev. D **79**, 084018 (2009) [arXiv:0812.2918 [hep-th]].

[22] T. Azeyanagi, N. Ogawa and S. Terashima, "The Kerr/CFT Correspondence and String Theory," Phys. Rev. D **79**, 106009 (2009) [arXiv:0812.4883 [hep-th]].

[23] Y. Nakayama, "Emerging AdS from extremally rotating NS5-branes," Phys. Lett. B **673**, 272 (2009) [arXiv:0812.2234 [hep-th]].

[24] H. Isono, T. S. Tai and W. Y. Wen, "Kerr/CFT correspondence and five-dimensional BMPV black holes," Int. J. Mod. Phys. A **24**, 5659 (2009) [arXiv:0812.4440 [hep-th]].

[25] J. J. Peng and S. Q. Wu, "Extremal Kerr black hole/CFT correspondence in the five dimensional Gódel universe," Phys. Lett. B **673**, 216 (2009) [arXiv:0901.0311 [hep-th]].

[26] C. M. Chen and J. E. Wang, "Holographic Duals of Black Holes in Five-dimensional Minimal Supergravity," Class. Quant. Grav. **27**, 075004 (2010) [arXiv:0901.0538 [hep-th]].

[27] F. Loran and H. Soltanpanahi, "5D Extremal Rotating Black Holes and CFT duals," Class. Quant. Grav. **26**, 155019 (2009) [arXiv:0901.1595 [hep-th]].

[28] A. M. Ghezelbash, "Kerr/CFT correspondence in the low energy limit of heterotic string theory," JHEP **0908**, 045 (2009) [arXiv:0901.1670 [hep-th]].

[29] H. Lu, J. w. Mei, C. N. Pope and J. F. Vazquez-Poritz, "Extremal static AdS black hole/CFT correspondence in gauged supergravities," Phys. Lett. B **673**, 77 (2009) [arXiv:0901.1677 [hep-th]].

[30] A. J. Amsel, G. T. Horowitz, D. Marolf and M. M. Roberts, "No Dynamics in the Extremal Kerr Throat," JHEP **0909**, 044 (2009) [arXiv:0906.2376 [hep-th]].

[31] G. Compere, K. Murata and T. Nishioka, "Central Charges in Extreme Black Hole/CFT Correspondence," JHEP **0905**, 077 (2009) [arXiv:0902.1001 [hep-th]].

[32] C. Krishnan and S. Kuperstein, "A Comment on Kerr-CFT and Wald Entropy," Phys. Lett. B **677**, 326 (2009) [arXiv:0903.2169 [hep-th]].

[33] K. Hotta, "Holographic RG flow dual to attractor flow in extremal black holes," Phys. Rev. D **79**, 104018 (2009) [arXiv:0902.3529 [hep-th]].

[34] D. Astefanesei and Y. K. Srivastava, "CFT Duals for Attractor Horizons," Nucl. Phys. B **822**, 283 (2009) [arXiv:0902.4033 [hep-th]].

[35] W. Y. Wen, "Holographic descriptions of (near-)extremal black holes in five dimensional minimal supergravity," arXiv:0903.4030 [hep-th].

[36] T. Azeyanagi, G. Compere, N. Ogawa, Y. Tachikawa and S. Terashima, "Higher-Derivative Corrections to the Asymptotic Virasoro Symmetry of 4d Extremal Black Holes," Prog. Theor. Phys. **122**, 355 (2009) [arXiv:0903.4176 [hep-th]].

[37] X. N. Wu and Y. Tian, "Extremal Isolated Horizon/CFT Correspondence," Phys. Rev. D **80**, 024014 (2009) [arXiv:0904.1554 [hep-th]].

[38] J. J. Peng and S. Q. Wu, "Extremal Kerr/CFT correspondence of five-dimensional rotating (charged) black holes with squashed horizons," Nucl. Phys. B **828**, 273 (2010) [arXiv:0911.5070 [hep-th]].

[39] J. D. Brown and M. Henneaux, "Central charges in the canonical realization of asymptotic symmetries: an example from three-dimensional gravity," Commun. Math. Phys. **104**, 207 (1986).

[40] T. Hartman and A. Strominger, "Central charge for AdS_2 quantum gravity," JHEP **0904**, 026 (2009) [arXiv:0803.3621 [hep-th]].

[41] M. Alishahiha and F. Ardalan, "Central charge for 2D gravity on AdS(2) and AdS(2)/CFT(1) correspondence," JHEP **0808**, 079 (2008) [arXiv:0805.1861 [hep-th]].

[42] A. Castro, D. Grumiller, F. Larsen and R. McNees, "Holographic description of AdS_2 black holes," JHEP **0811**, 052 (2008) [arXiv:0809.4264 [hep-th]].

[43] V. Balasubramanian, J. de Boer, M. M. Sheikh-Jabbari and J. Simon, "What is a chiral 2d CFT? And what does it have to do with extremal black holes?," JHEP **1002**, 017 (2010) [arXiv:0906.3272 [hep-th]].

[44] A. Castro, C. Keeler and F. Larsen, "Three Dimensional Origin of AdS_2 Quantum Gravity," arXiv:1004.0554 [hep-th].

[45] A. Castro, A. Maloney and A. Strominger, "Hidden Conformal Symmetry of the Kerr Black

Hole," arXiv:1004.0996 [hep-th].

[46] C. Krishnan, "Hidden Conformal Symmetries of Five-Dimensional Black Holes," arXiv:1004.3537 [hep-th].